Systems Innovation 3
– System Modeling –

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What is model?

Fashion model

Plastic model

Model and simulation

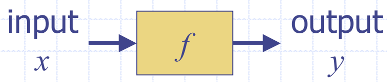
- What is a model?
  - Relevant features of world entities extracted by discarding irrelevant features and represented in a particular form, when one tries to understand happenings in the world
- Simulation
  - Methodology to elucidate, understand, and predict behaviour of objects by manipulating models without decreasing complexity or wholeness of the reality

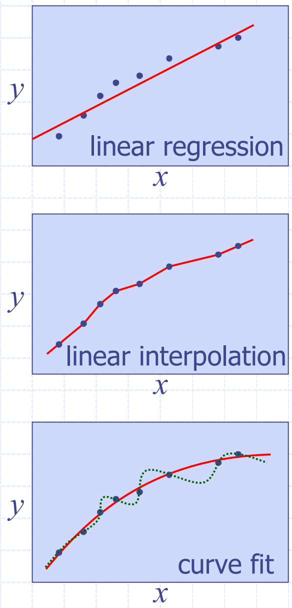
Approaches for system modeling

- static v.s. dynamic
- linear v.s. non-linear
- steady state v.s. transient
- open v.s. closed
- continuous v.s. discrete
- deterministic v.s. probabilistic
- qualitative (symbolic) v.s. quantitative

Static model

◆Function model





◆Least square fit

$$\sum_i w_i [f(x_i) - y_i]^2 \rightarrow \min$$

Sophisticated static model

◆Multivariate statistical analysis

- Regression analysis
- Principal component analysis
- Factor analysis
- Cluster analysis
- Quantification method 1,2,3,4
- Covariance analysis
- Multi-dimensional scaling
- Conjoint analysis

Factor analysis

◆Linear model

$$
\begin{cases}
z_{i1} = a_{11} f_{i1} + a_{12} f_{i2} + \cdots + a_{1m} f_{im} + u_{i1} \\
z_{i2} = a_{21} f_{i1} + a_{22} f_{i2} + \cdots + a_{2m} f_{im} + u_{i2} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
z_{in} = a_{n1} f_{i1} + a_{n2} f_{i2} + \cdots + a_{nm} f_{im} + u_{in}
\end{cases}
$$

$$(i = 1, 2, \cdots, p)$$

unique component

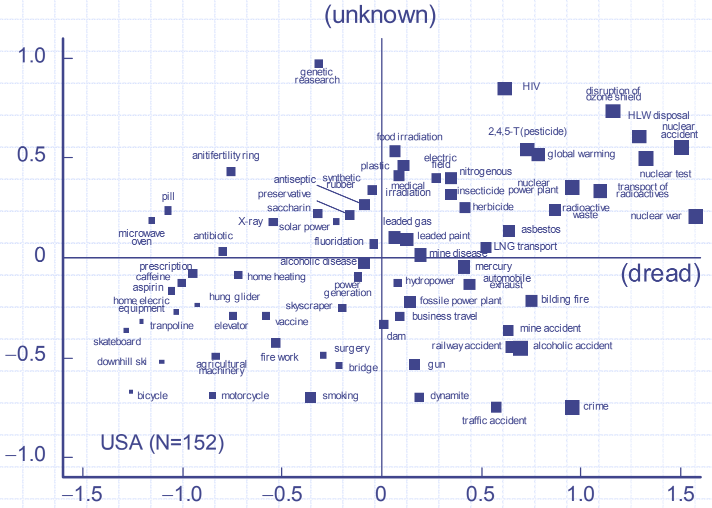
data vector

common factor

◆Obtain a_{ij} that minimizes $e^2 = \sum_{j=1}^{n} \frac{1}{p} \sum_{i=1}^{p} u_{ij}^2$

Example of statistic model

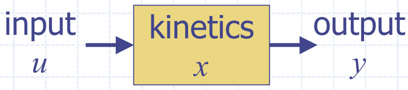
– Factor analysis of public risk image –



Kinetics model

◈State space representation

$$
\begin{cases}
\dfrac{dx_1}{dt} = f_1(x_1, x_2, \cdots, x_n, u) \\
\dfrac{dx_2}{dt} = f_2(x_1, x_2, \cdots, x_n, u) \\
\cdots\cdots\cdots\cdots\cdots\cdots \\
\dfrac{dx_n}{dt} = f_n(x_1, x_2, \cdots, x_n, u)
\end{cases}
$$



$$
\begin{cases}
y_1 = g_1(x_1, x_2, \cdots, x_n) \\
y_2 = g_2(x_1, x_2, \cdots, x_n) \\
\cdots\cdots\cdots\cdots\cdots\cdots \\
y_m = g_m(x_1, x_2, \cdots, x_n)
\end{cases}
$$

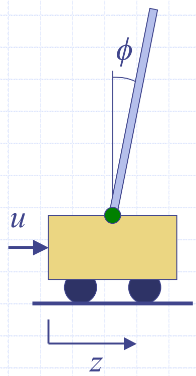
- If f, g are linear in terms of x_i, linear model.

Example of kinetics model
– Classical dynamics –

◈Pole and cart system

$x_1 = z \quad x_2 = \phi \quad x_3 = \dot{z} \quad x_4 = \dot{\phi}$

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\varsigma & 0 \\ 0 & \alpha g & \alpha\varsigma & -\beta\eta \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \xi \\ -\alpha\xi \end{bmatrix} u
$$

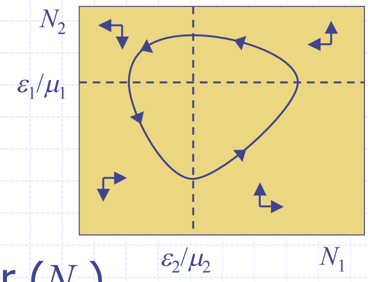


Example of kinetics model
– Ecosystem –

◆Lotka-Volterra model

$$\dot{N}_1 = (\varepsilon_1 - \mu_1 N_2) N_1$$

$$\dot{N}_2 = (-\varepsilon_2 + \mu_2 N_1) N_2$$

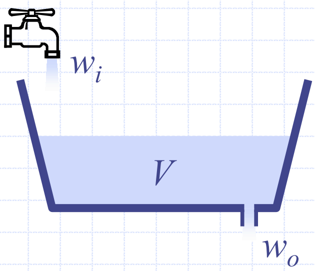


◆Prey (N_1) and predator (N_2)

- Predator hunts and eats prey.
- Predator cannot survive without prey.
- Predator breeds rapidly, if prey is affluent.

System dynamics (1)
– Diagramatic representation of kinetics –

Bathtub model



$$\frac{dV}{dt} = w_i - w_o$$

$$V = \int (w_i - w_o) dt$$

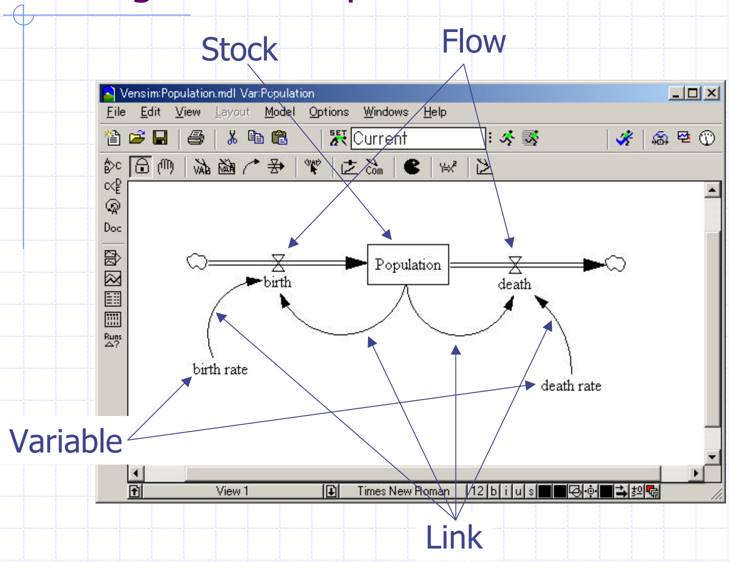
Stock (V): reserve、population, stock, ...

Flow (w): inflow-outflow, birth-death, income-expenditure, ...

System dynamics (2)
– Diagramatic representation of kinetics –

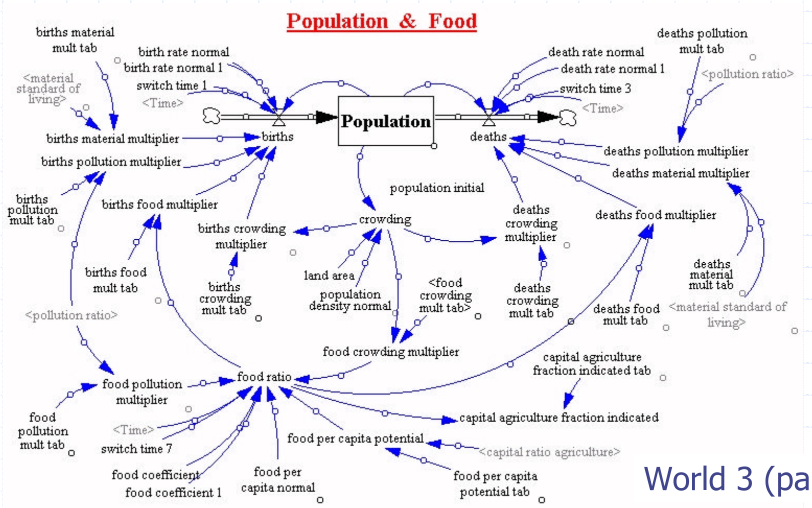
- J.W.Forrester (1956)
- Modeling tool
  - Stocks, flows, other variables, links
  - Diagramatic representation of causal relations
  - Model construction using GUI
- Simulation tool
  - Numerical solution of ODEs
  - Supporting functions for setting simulation scenarios and for visualizing simulation results

System dynamics (3)
– Diagramatic representation of kinetics –



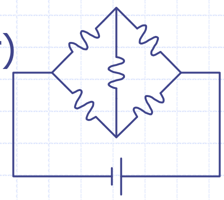
System dynamics (4)

– World model –



World 3 (part)

Network model



- Node (control volume, reservoir)
  - Uniform property and state
  - Internal kinetics (state equation)
- Link (junction)
  - Connection between nodes
  - Flow is defined
- Kirchhoff's law
  - Net flow into each junction is zero.
  - Driving force balances with pressure drop along each circuit.

$$\begin{cases} \sum I_i = 0 \\ \sum R_i I_i = E \end{cases}$$

Example of network model
– Cooling system of a nuclear power plant –

◆ Dynamics in each node

Conservation of mass

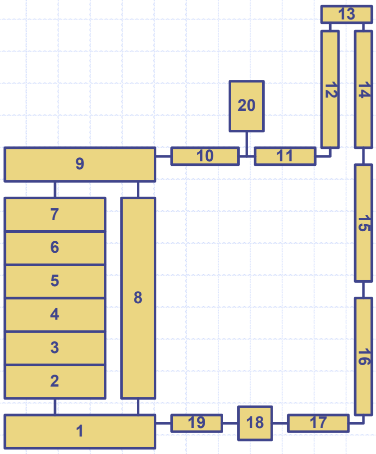
$$V_i \frac{d\rho_i}{dt} = w_{in,i} - w_{out,i}$$

Conservation of energy

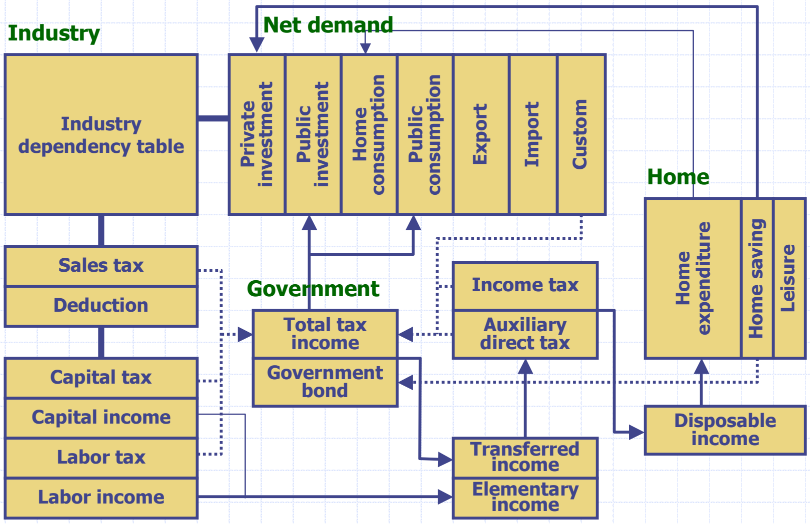
$$V_i \frac{dh_i}{dt} = h_{in,i} w_{in,i} - h_i w_{out,i} + Q_i$$

State equation

$$\frac{d\rho_i}{dt} = \frac{\partial \rho_i}{\partial h_i}\frac{dh_i}{dt} + \frac{\partial \rho_i}{\partial P_i}\frac{dP_i}{dt}$$



Example of network model
– Money flow of national economy –



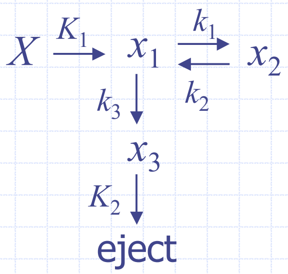
Equilibrium (balance) model

- Obtained by setting $dx_i/dt=0$ in kinetics model
  - Stable or unstable equilibrium point
  - Sometimes called static or adiabatic model

$$
\begin{cases}
\dfrac{dx_1}{dt} = f_1(x_1, x_2, \cdots, x_n, u) = 0 \\
\dfrac{dx_2}{dt} = f_2(x_1, x_2, \cdots, x_n, u) = 0 \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\dfrac{dx_n}{dt} = f_n(x_1, x_2, \cdots, x_n, u) = 0
\end{cases}
$$

Example of equilibrium model

– Physical or chemical reaction –



$$
\begin{cases}
K_1(X - x_1) + k_2 x_2 = k_1 x_1 + k_3 x_1 \\
k_1 x_1 = k_2 x_2 \\
k_3 x_1 = K_2 x_3
\end{cases}
$$

$$
x_1 = \frac{K_1 X}{K_1 + k_3} \quad x_2 = \frac{k_1}{k_2} x_1 \quad x_3 = \frac{k_3}{K_2} x_1
$$

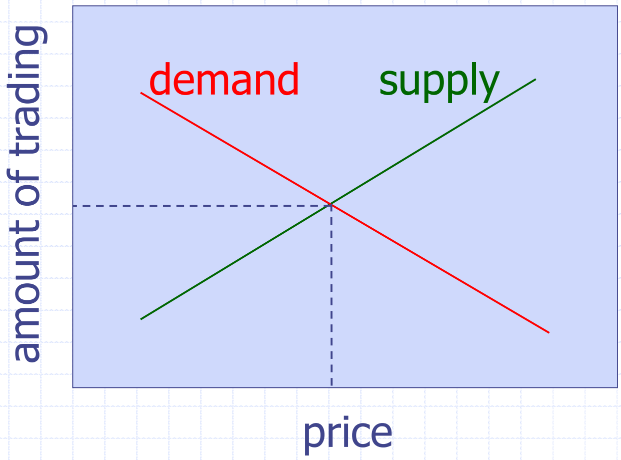
αA + βB + … $\Leftrightarrow$ γC + δD + …

$$
\frac{[C]^{\gamma}[D]^{\delta}\cdots}{[A]^{\alpha}[B]^{\beta}\cdots} = K
$$

Law of mass action

Example of equilibrium model

– Fundamental economics of market –



Continuum model

– Diffusion of matter –

$$\delta x \delta y \frac{du}{dt} = \delta y (J_{x-} - J_{x+}) + \delta y (J_{y-} - J_{y+}) + \delta x \delta y S$$

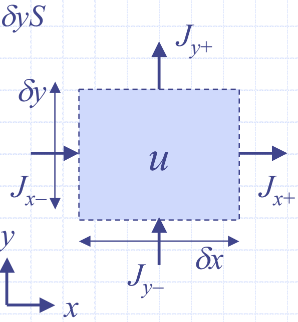
$$\frac{du}{dt} = -\left(\frac{J_{x+} - J_{x-}}{\delta x} + \frac{J_{y+} - J_{y-}}{\delta y} \right) + S$$

$$\delta x, \delta y \to 0$$

$$\frac{du}{dt} = -\frac{\partial J_x}{\partial x} - \frac{\partial J_y}{\partial y} + S$$

$$J_x = -D \frac{\partial u}{\partial x} \qquad J_y = -D \frac{\partial u}{\partial y}$$

$$\frac{du}{dt} = D \frac{\partial^2 u}{\partial x^2} + D \frac{\partial^2 u}{\partial y^2} + S$$



u : concentration
(matter, momentum, heat)

$\mathbf{J}$: flow vector

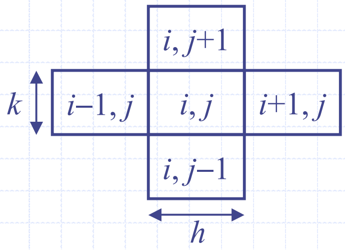
Lattice (grid) model

– Finite difference method –

$$D\frac{\partial^2 u}{\partial x^2} + D\frac{\partial^2 u}{\partial y^2} + S = 0$$

$$\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i-1,j} + u_{i+1,j} - 2u_{i,j}}{h^2}$$

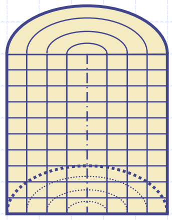
$$\frac{\partial^2 u}{\partial y^2} \approx \frac{u_{i,j-1} + u_{i,j+1} - 2u_{i,j}}{k^2}$$



$$-au_{i,j-1} - bu_{i-1,j} + cu_{i,j} - du_{i+1,j} - eu_{i,j+1} = f_{i,j}$$

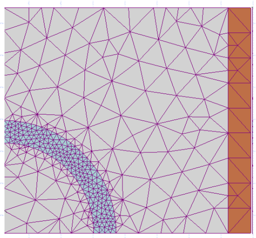
$$a = \frac{h}{k}D \quad b = \frac{k}{h}D \quad d = \frac{k}{h}D \quad e = \frac{h}{k}D$$

$$c = a + b + d + e \quad f_{i,j} = hkS_{i,j}$$



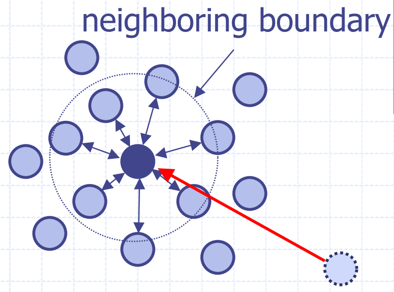
Finite element model

- Discretize continuous objects in the system into many finite meshes of an elementary shape.
  - Triangular or rectangular shells / columns (2D)
  - Hexahedral or tetrahedral meshes (3D)
- Values of a physical parameter within a mesh are represented with some basis function and values at the mesh corners.
  - Piecewise linear or quadratic basic functions
- A weak form of the differential equation is set up for the set of corner values over the entire finite elements.
- The derived set of linear equations is solved numerically.



Particle model

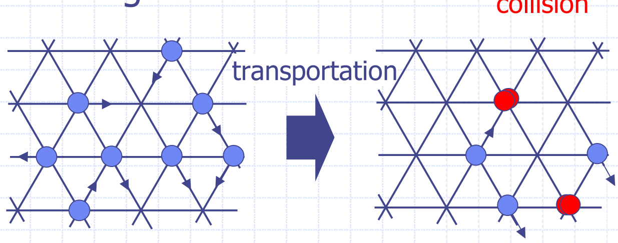
- Particle
  - Molecule, atom, etc.
  - Small volume of fluid
  - Virtual particle
- Interaction (collision)
  - Local v.s. global
  - Deterministic v.s. probabilistic



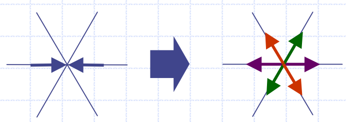
- Transportation process
  - Translatory motion in space
- Collision process
  - Interaction with neighboring particles

Example of particle model
– Lattice Gas Automaton (LGA) –

2-D triangular lattice



Collision rules



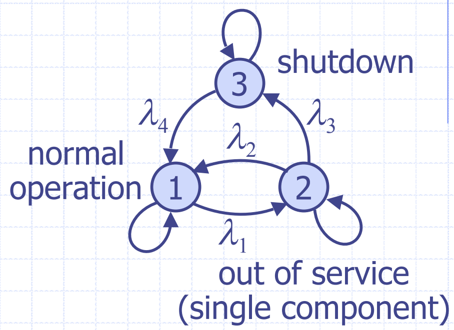
Coarse graining

$$\rho(\mathbf{x},t) = \sum_{i=1}^{6} \langle n_i(\mathbf{x},t) \rangle$$

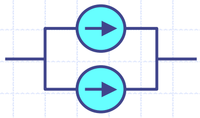
$$\mathbf{u}(\mathbf{x},t) = \sum_{i=1}^{6} \mathbf{c}_i \langle n_i(\mathbf{x},t) \rangle / \rho$$

Markov model

- Finite system states
- State transition



Redundant standby system

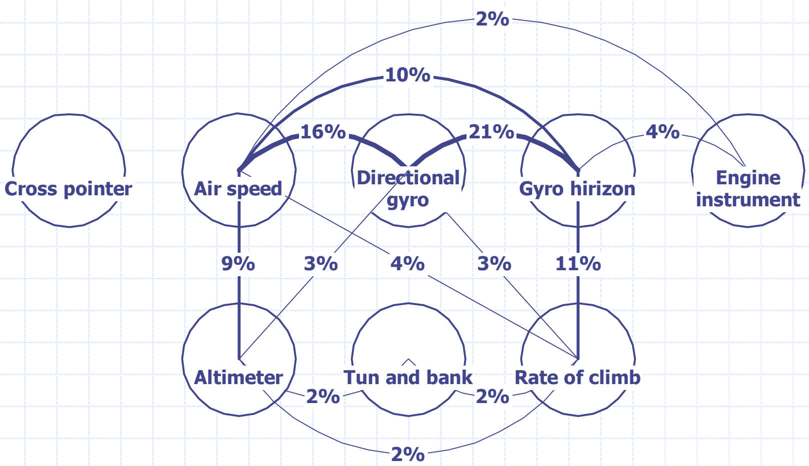


Kolmogorov equations

$$\begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \end{bmatrix} = \begin{bmatrix} -\lambda_1 & \lambda_2 & \lambda_4 \\ \lambda_1 & -(\lambda_2 + \lambda_3) & 0 \\ 0 & \lambda_3 & -\lambda_4 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

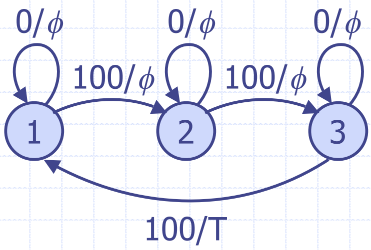
Example of Markov model

– Eye-movement between aircraft instruments –



Automaton model

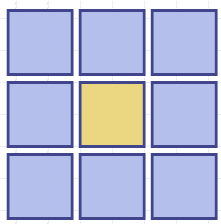
- Automatic vendor that sells tickets of 300 JPY but accepts only 100 JPY coins



States:	A={1, 2, 3}
Inputs:	B={0, 100}
Outputs:	C={ϕ, T}
State transition:	A x B $\rightarrow$ A
Output function:	A x B $\rightarrow$ C

Cellular automaton model

- Many uniform cells are arranged on a regular lattice.
- Each cell takes any of predefined finite states.
- Every cell changes its state at each time interval.
- The new state depends on the previous state and the states of neighboring cells.

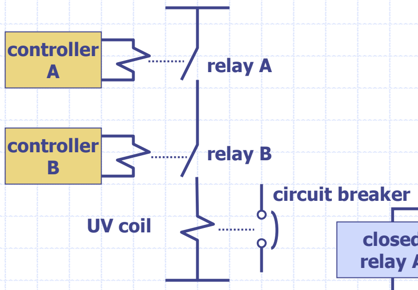


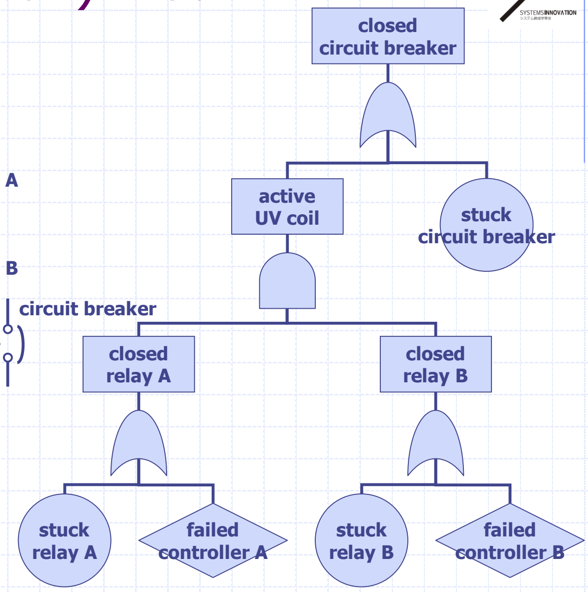
Moore's neighborhood

- Conway's game of life
  - Any live cell with two or three live neighbors survives.
  - Any dead cell with exactly three live neighbors comes to life.

Logic (Boolean) model

– Fault tree –





- Boolean algebra
  - Truth value (T/F)
  - Negation (NOT)
  - Conjunction (AND)
  - Disjunction (OR)

Symbolic model

– Production system –

- Context free grammar (Schank)
  - sentence → verb, noun phrase
  - noun phrase → article, adjective, noun
  - noun phrase → article, adjective, noun, preposition, noun
  - verb → 'erase'
  - article → 'the'
  - adjective → 'last'
  - noun → 'word'

  ....

Symbolic (Formal) logic model
– Predicate calculus –

◆ Formal representation (syntax)

$P(t_1, \ldots, t_n)$	Relation P holds among $t_1, \ldots t_n$
$\forall x\ P(x)$	$P(x)$ holds for all x
$\exists x\ P(x)$	Some x exists such that $P(x)$ holds
$\neg\ \wedge \vee \rightarrow$	Operators

◆ Formal inference (resolution)

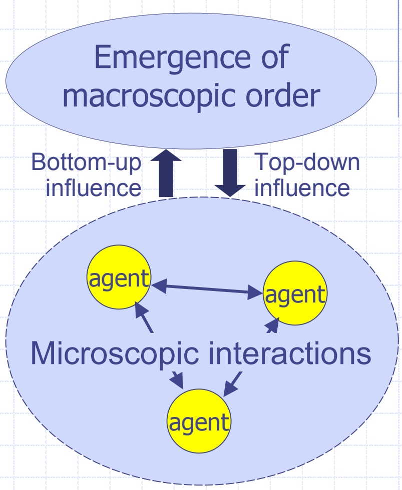
$\forall x \forall y \forall z$ parent$(x, y) \wedge$parent$(y, z) \rightarrow$ grandparent(x, z)
parent(Namihei, Sazae)
parent(Sazae, Tara) — axiom

grandparent(Namihei, Tara) — theorem

Agent model

◆ Agent has ability to …
- select its own action
- collect or exchange information
- adapt itself to surrounding situation

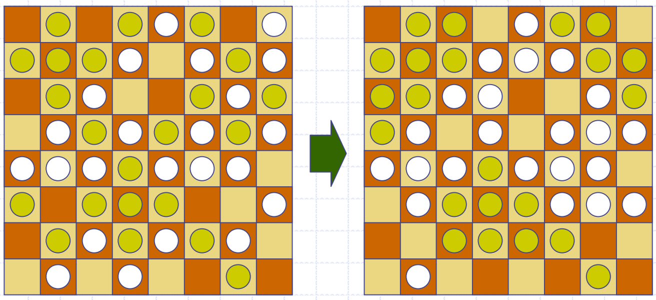
◆ Multi-agent system
- System that achieves the designated goal by cooperative action of many agents



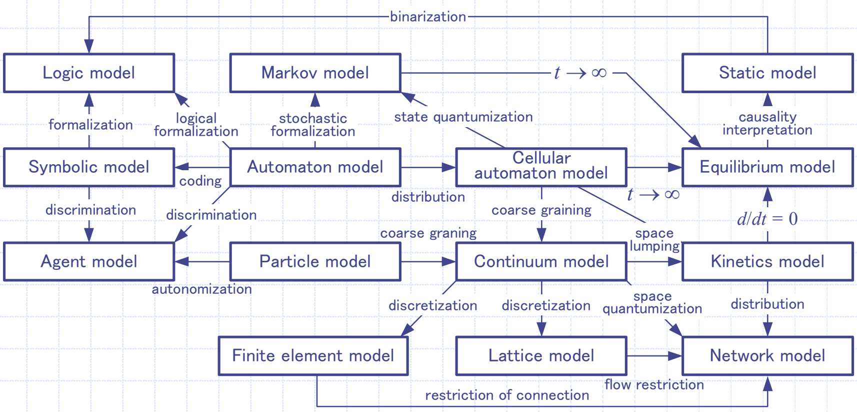
Micro-macro link

Example of agent model
- Shelling's model of segregation -

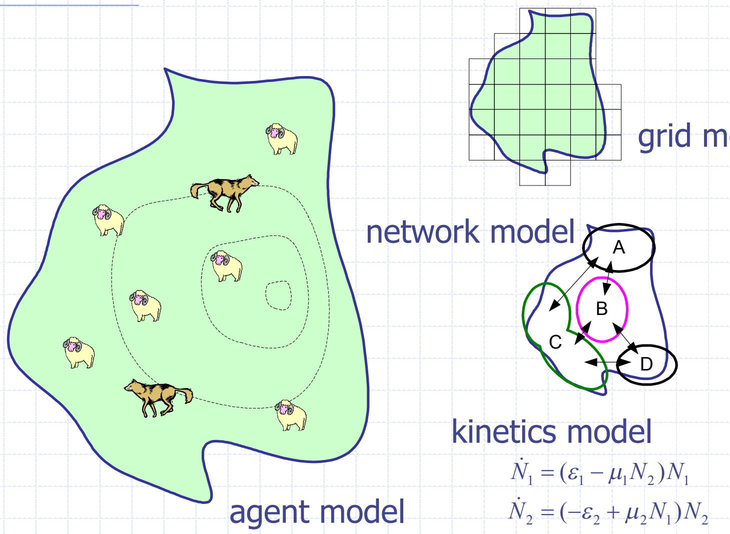
- ◆ Organization of local ethnic community
  - A very weak preference for the own ethnicity can promote segregation



Interrelations between various models



Grazing in a uninhabited island



Points in system modeling

- You can apply almost any style of modeling to each system if you wish.
- Each style gives you a particular view of the target system.
- Which style is appropriate depends on the purpose of modeling (what you want to see by modeling).